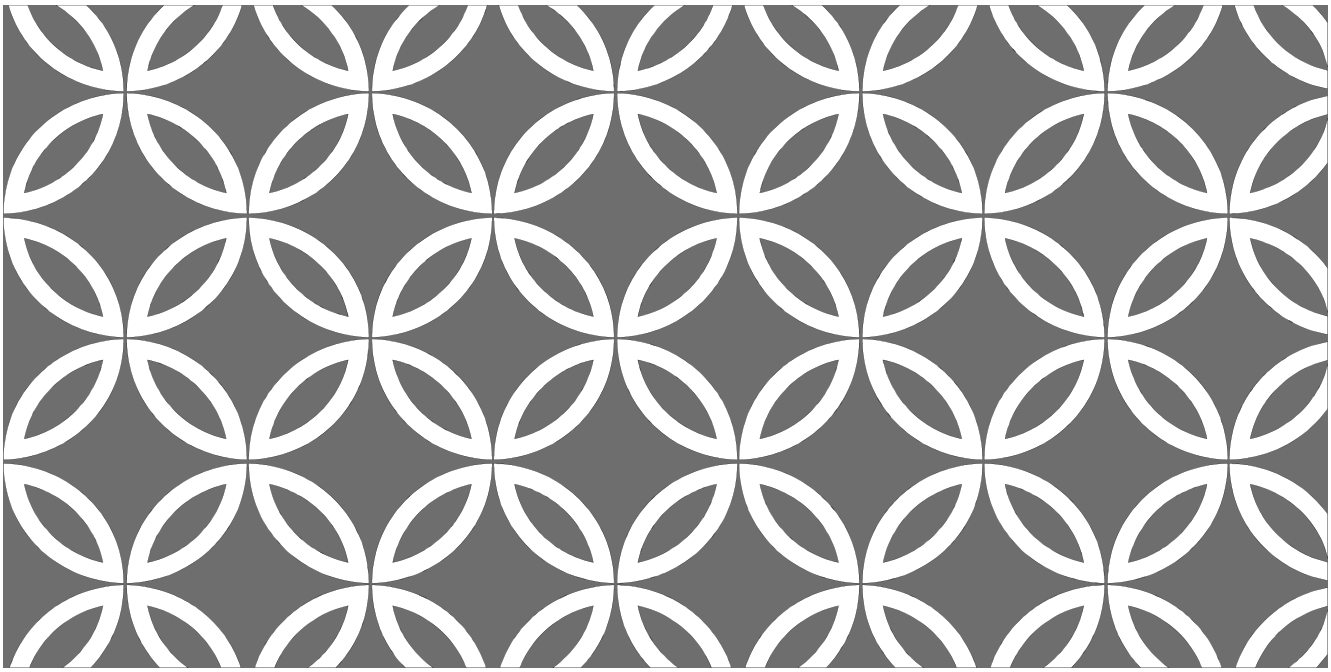


LINEAR CIRCUIT ANALYSIS (EED) – U.E.T. TAXILA | 03

ENGR. M. MANSOOR ASHRAF

INTRODUCTION

On the basis of Ohm's law and Kirchhoff's laws, the two well known circuit analysis techniques may be developed.

These techniques are: Nodal Analysis and Mesh Analysis.

Nodal Analysis is based on systematic application of KCL.

Mesh Analysis is based on systematic application of KVL.

NODAL ANALYSIS

In this technique, the node voltages are computed by KCL of 'n' node circuit without voltage source.

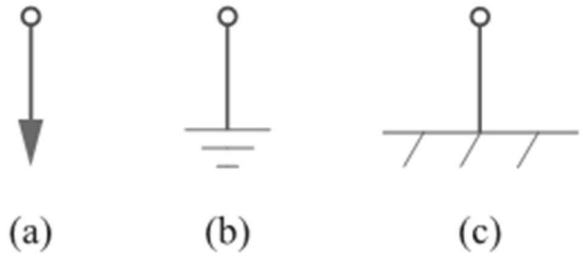
Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

NODAL ANALYSIS

The first step in nodal analysis is selecting a reference node which is also called ground having zero potential.

A reference node is indicated by any of the three symbols.



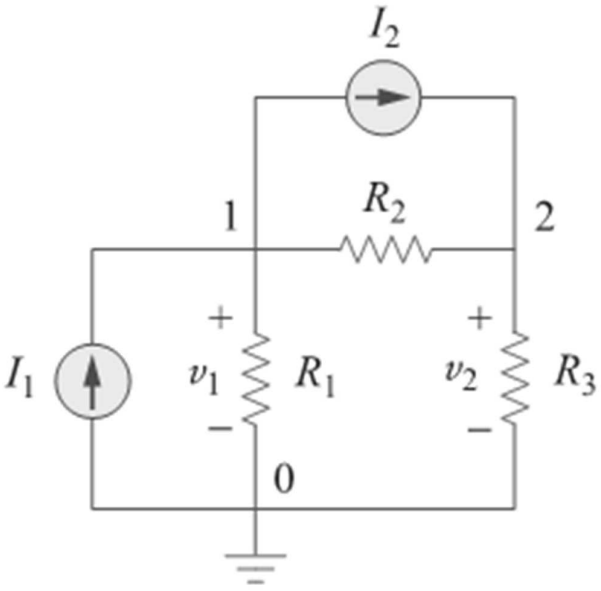
If there are 'n' nodes in a circuit, after selecting reference node, the circuit is left with (n-1) nodes.

NODAL ANALYSIS

In the circuit shown, node '0' is reference node while nodes '1' and '2' are non-reference nodes.

Keep in mind that the node voltages are always defined with respect to the reference node.

The second step in nodal analysis is to apply KCL for each non-reference node in the circuit.



NODAL ANALYSIS

At node 1;

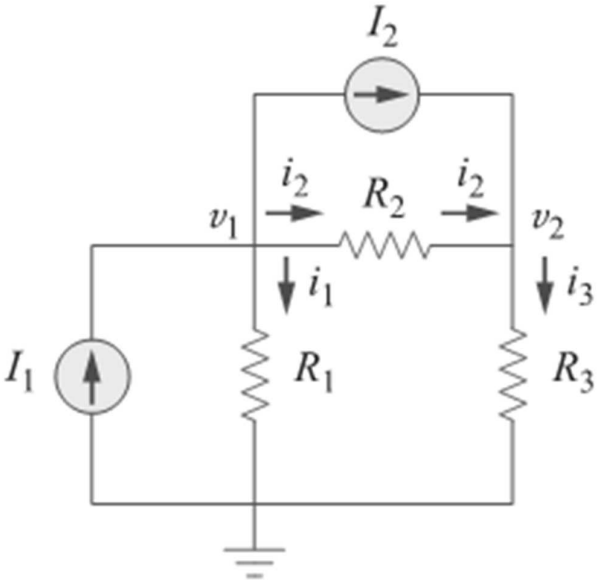
$$I_1 = I_2 + i_1 + i_2$$

At node 2;

$$I_2 + i_2 = i_3$$

Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$



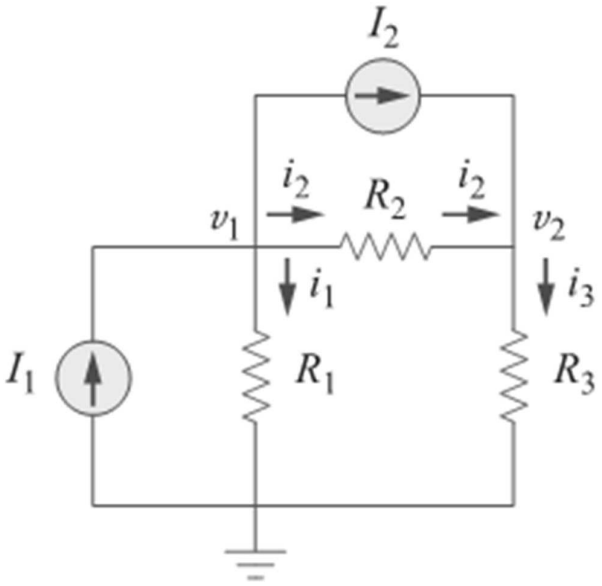
NODAL ANALYSIS

Express the branch currents in terms of voltage by application of Ohm's law.

$$i_1 = \frac{v_1 - 0}{R_1}$$

$$i_3 = \frac{v_2 - 0}{R_3}$$

$$i_2 = \frac{v_1 - v_2}{R_2}$$



NODAL ANALYSIS

The third step in nodal analysis is to solve system of simultaneous equations to obtain node voltages.

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

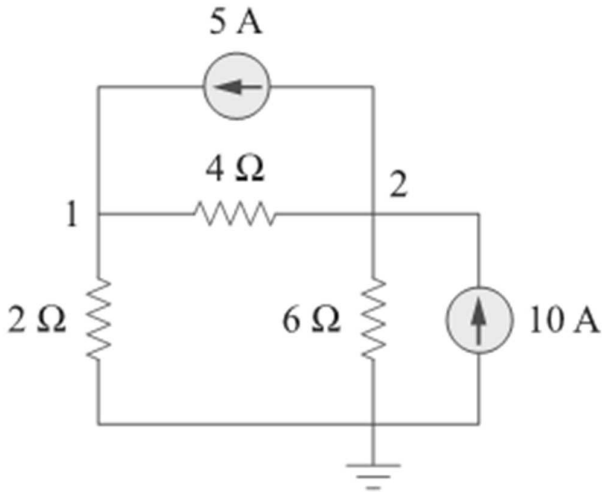
$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2)$$

$$I_2 + G_2(v_1 - v_2) = G_3 v_2$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

PROBLEMS

Calculate the node voltages in the circuit?



(13.33 V, 20 V)

NODAL ANALYSIS WITH VOLTAGE SOURCES

Case 1: If a voltage source is connected between reference node and non-reference node, the voltage of that non-reference node will be equal to the voltage of voltage source.

Case 2: If the voltage source is connected between two non-reference nodes, the two non-reference nodes form a generalized node or super node.

A Super Node is formed by enclosing a voltage source connected between two non-reference nodes and any element connected in parallel with it.

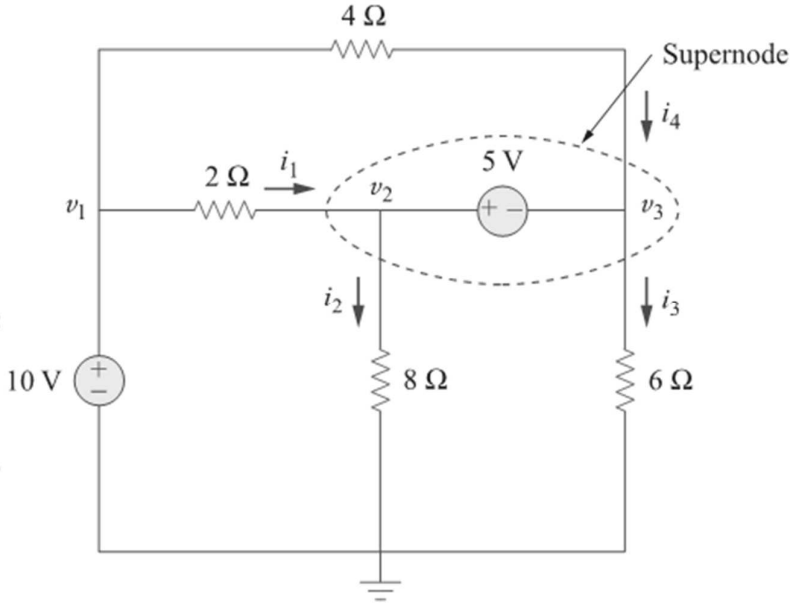
NODAL ANALYSIS WITH VOLTAGE SOURCES

If super node exists in a circuit, both KCL and KVL are applied to determine node voltages.

Applying KCL;

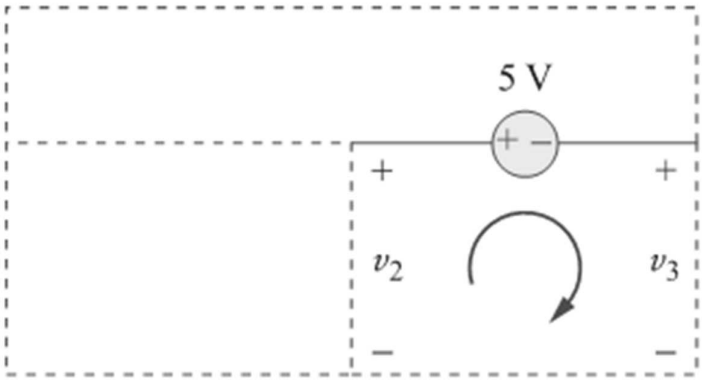
$$i_1 + i_4 = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$



NODAL ANALYSIS WITH VOLTAGE SOURCES

Applying KVL

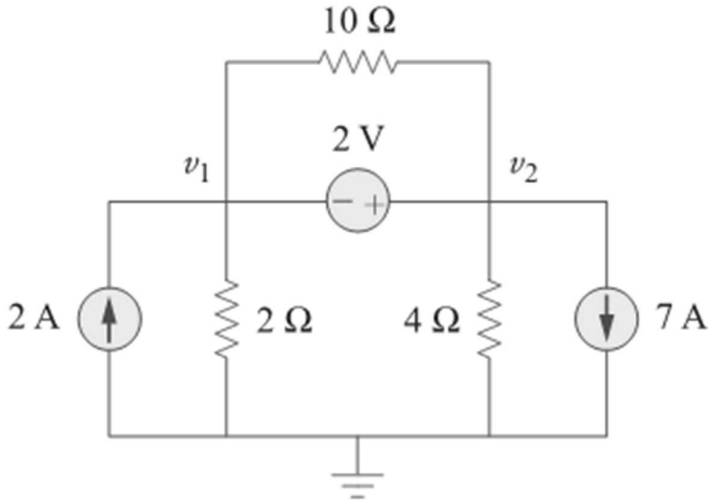


$$-v_2 + 5 + v_3 = 0$$

$$v_2 - v_3 = 5$$

PROBLEMS

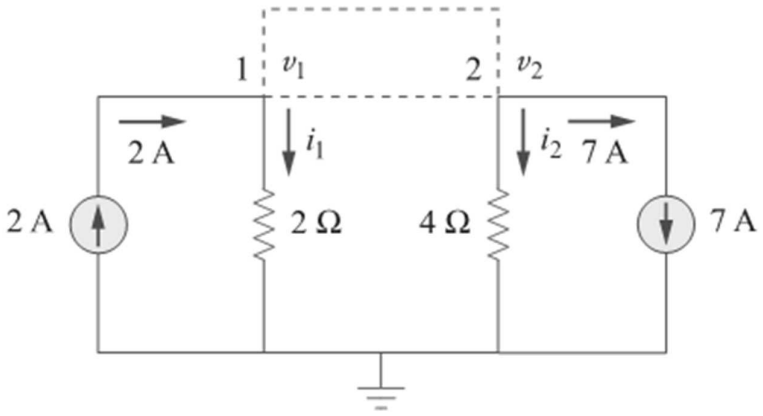
Calculate the node voltages in the circuit?



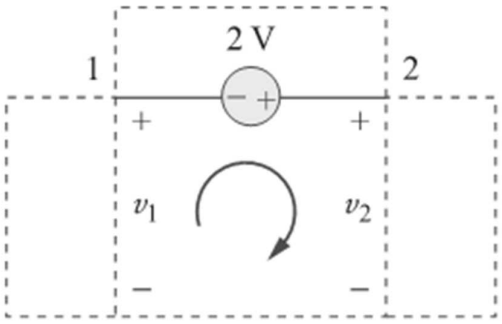
(-7.33 V, -5.33 V)

PROBLEMS

For KCL

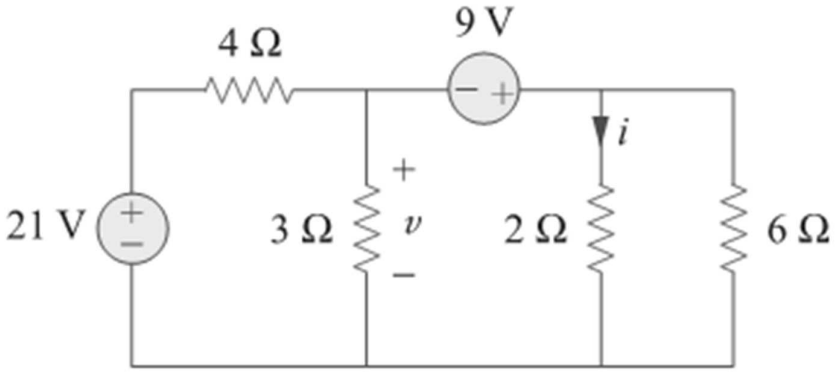


For KVL



PROBLEMS

Calculate the node voltages in the circuit?



(-0.6 V, 4.2 A)

MESH ANALYSIS

Mesh analysis is based on systematic application of KVL.

Loop is a closed path with no node passed more than once.

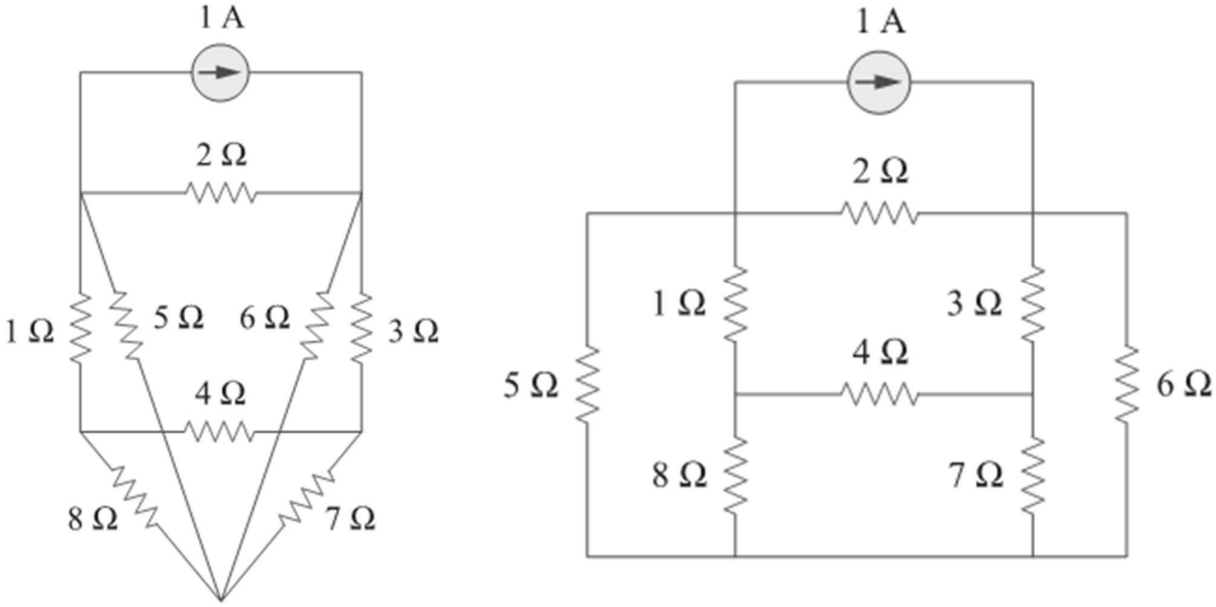
A Mesh is a loop which does not contain any other loops within it.

Mesh analysis is only applicable to a circuit that is planar.

A Planar Circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is non-planar.

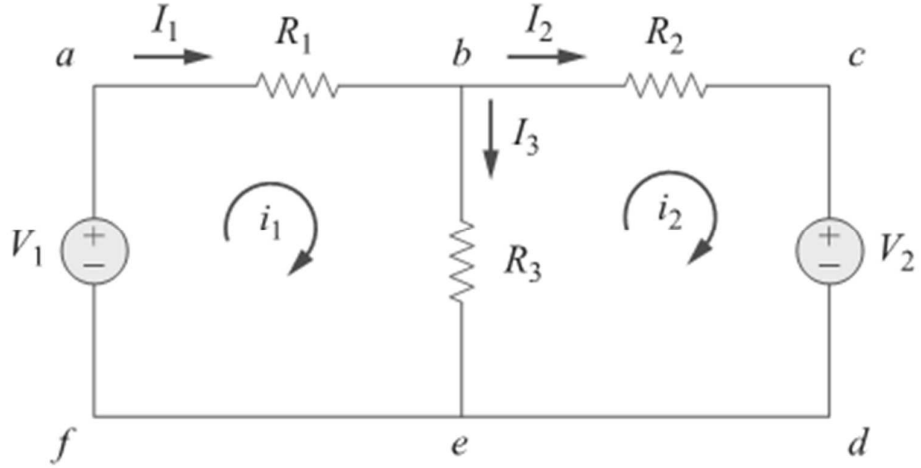
MESH ANALYSIS

A non-planar circuit may be converted into planar circuit after redrawing it.



MESH ANALYSIS

Consider the circuit;



The paths *abefa* and *bcdeb* are meshes, but path *abcdefa* is not a mesh.

MESH ANALYSIS

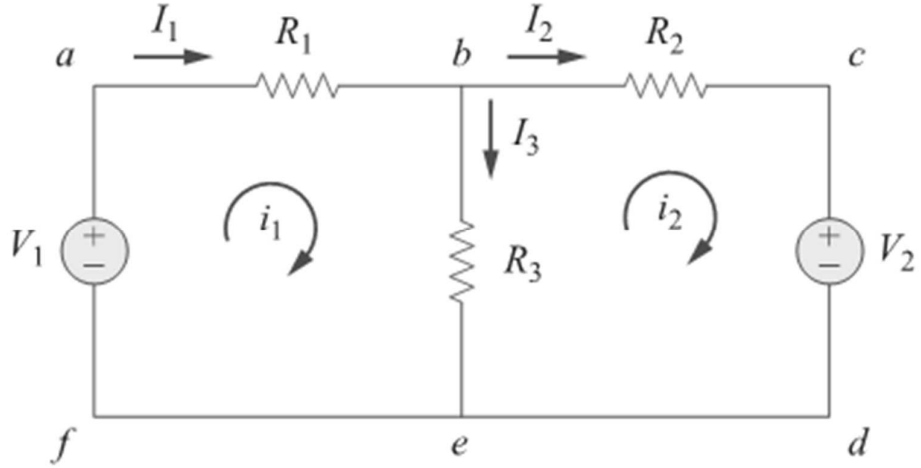
The steps to apply mesh analysis without current sources to planar circuits are;

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

MESH ANALYSIS

The 1st step in mesh analysis is to assign the mesh currents.



The 2nd step is to apply KVL to each mesh.

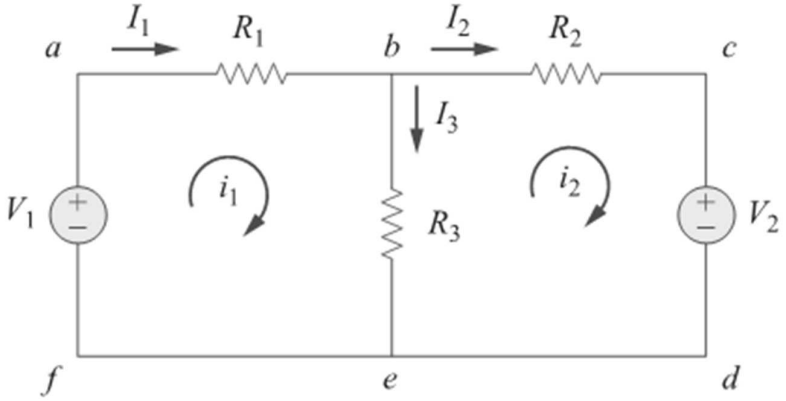
MESH ANALYSIS

Mesh 1: $-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

Mesh 2: $R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$



MESH ANALYSIS

The 3rd step is to solve system of simultaneous equations to find mesh currents.

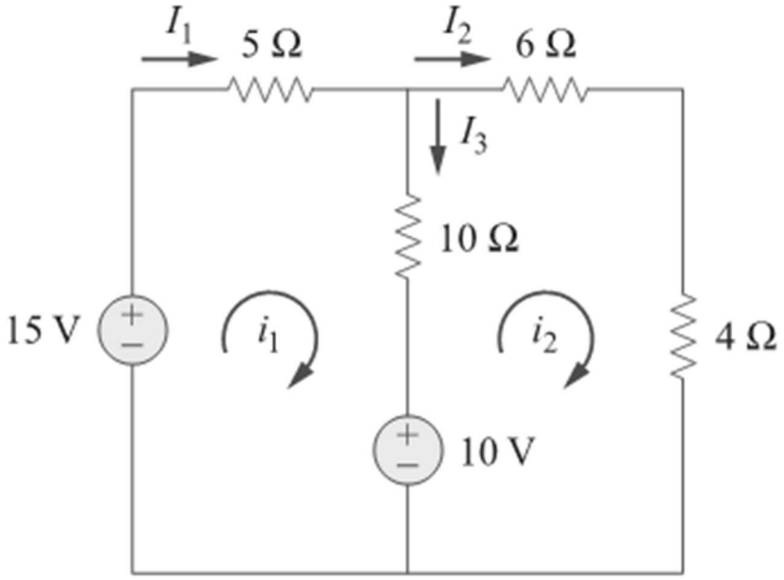
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

The branch currents and mesh currents may be equal or different.

$$I_1 = i_1, \qquad I_2 = i_2, \qquad I_3 = i_1 - i_2$$

PROBLEMS

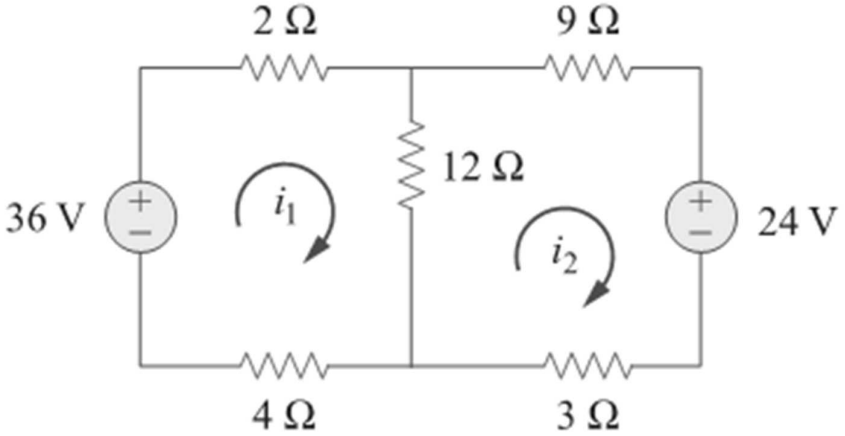
Calculate the branch currents in the circuit?



(1 A, 1 A, 0 A)

PROBLEMS

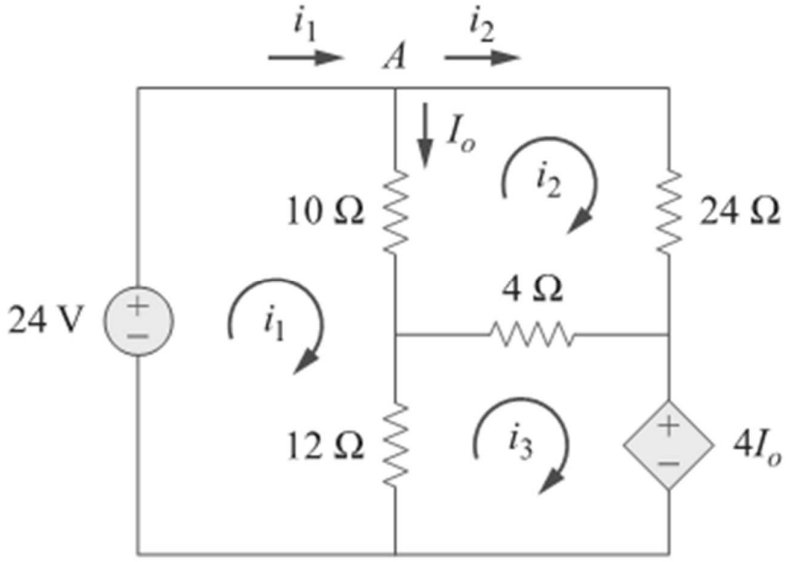
Calculate the mesh currents in the circuit?



(2 A, 0 A)

PROBLEMS

Calculate the current in the circuit?



(1.5 A)

MESH ANALYSIS WITH CURRENT SOURCES

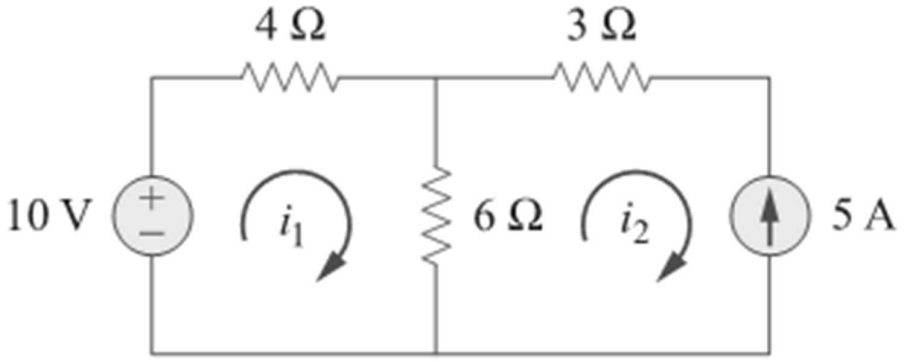
Case 1: When a current source exists only in one mesh, the mesh current will be equal to the current of that current source.

Case 2: When a current source exists between two meshes, the two meshes will behave as single mesh which is called super mesh.

A Super Mesh results when two meshes have a current source in common.

When super mesh exists in a circuit, both KVL and KCL are applied.

MESH ANALYSIS WITH CURRENT SOURCES

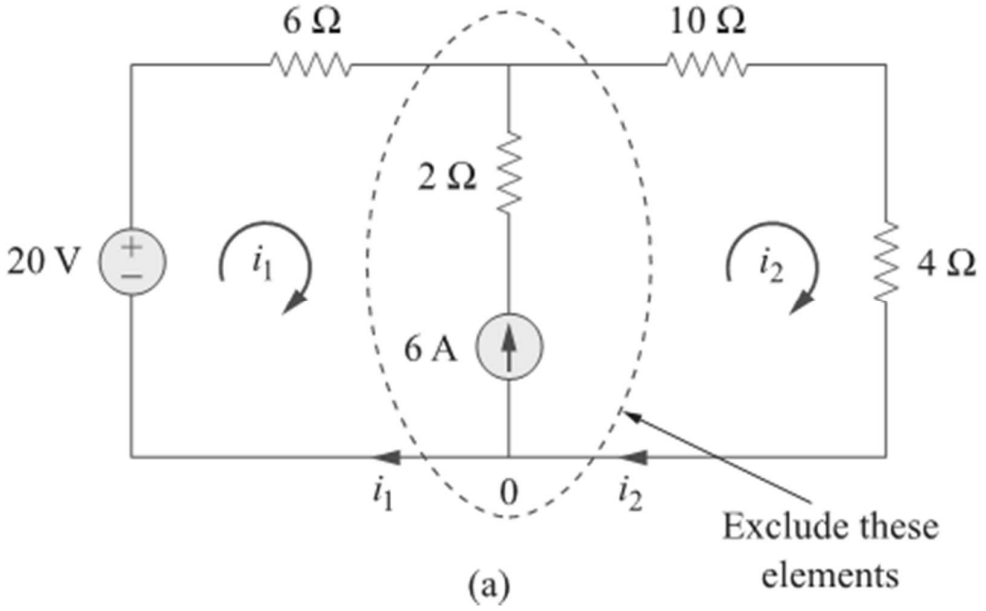


$$i_2 = -5$$

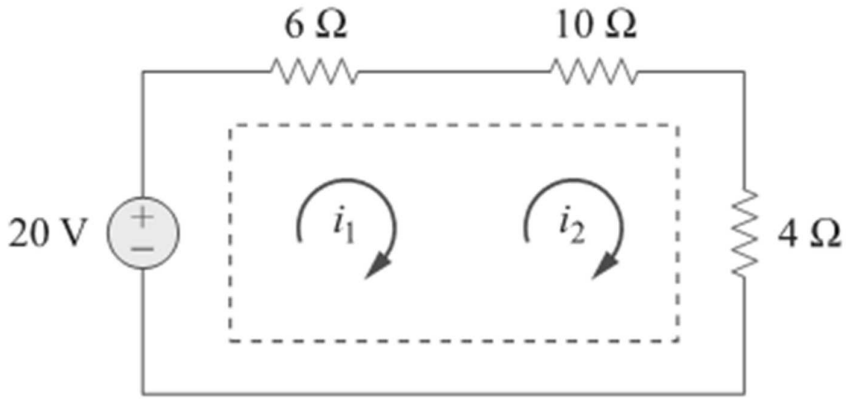
$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

$$i_1 = -2 \text{ A}$$

MESH ANALYSIS WITH CURRENT SOURCES



MESH ANALYSIS WITH CURRENT SOURCES

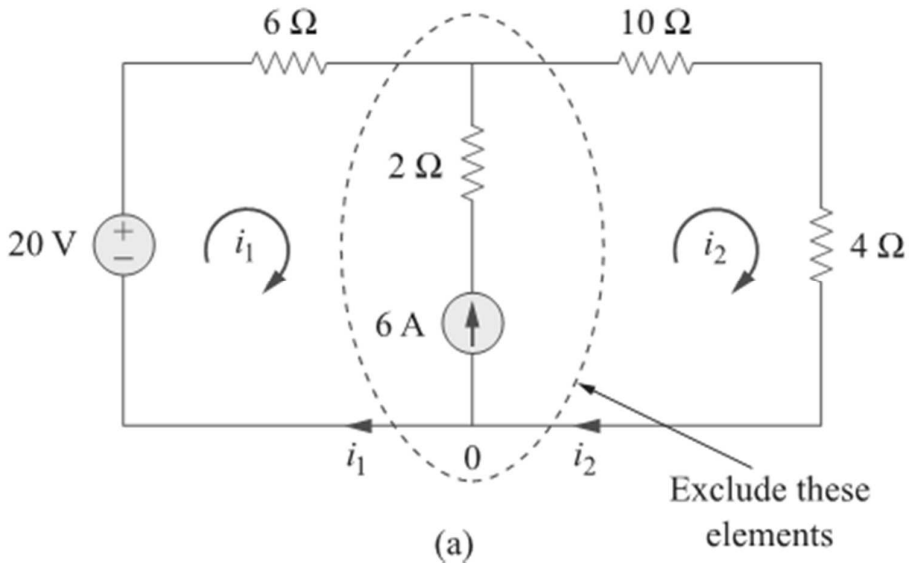


$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20$$

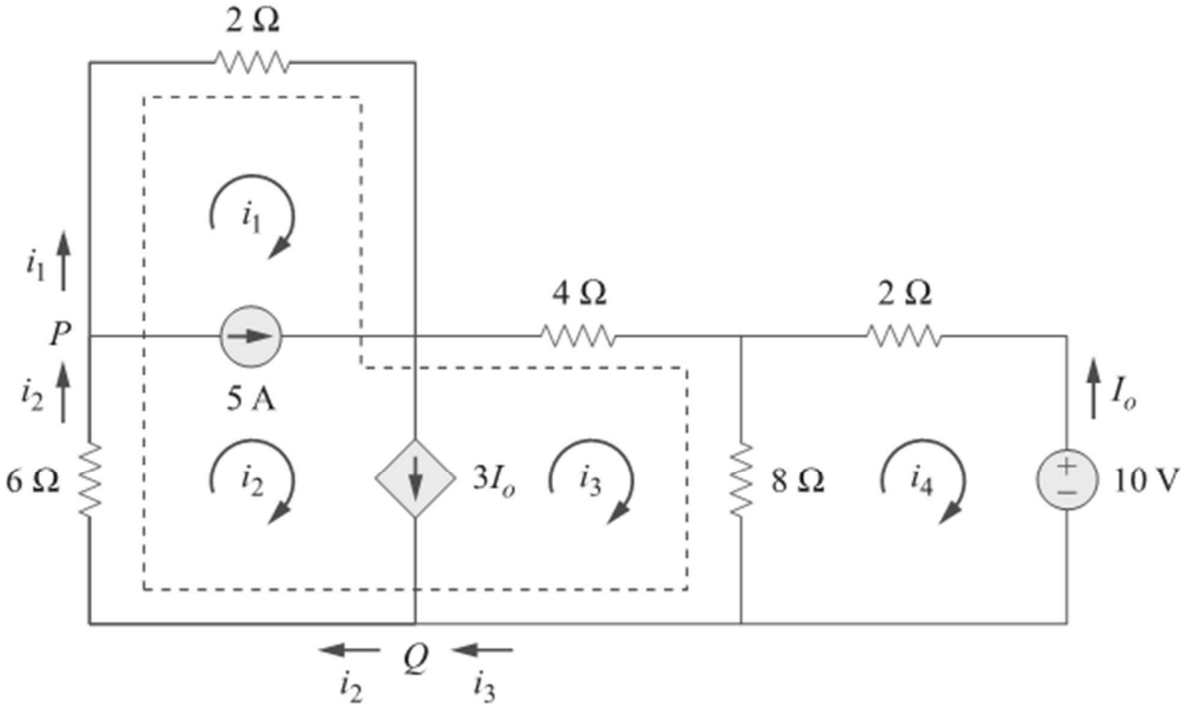
MESH ANALYSIS WITH CURRENT SOURCES

Apply KCL at node 0; $i_2 = i_1 + 6$



PROBLEMS

Calculate currents in the circuit? (-7.5, -2.5, 3.93, 2.14)



REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 03 – Methods of Analysis (3.1 – 3.7)

Exercise Problems: 3.1 – 3.74

Do exercise problem yourself.